**Unit 2: Analytic Geometry**

**Lesson 3: Apply Slope, Midpoint and Length Formulas**

Example 1: Find the Shortest Route

1. A ranger cabin is to be built in a flat wooded area near the straight road that connects the two campgrounds in a park. A new side road will connect the cabin to the campground road. One the park map, the campgrounds have coordinates A (2.0, 8.5) and B (10.0, 4.5), while the site for the cabin is at R (6.0, 1.5). Each unit on the map grid represents 500 m

A (2.0, 8.5)

B (10, 4.5)

R (6.0, 1.5)

* 1. Find the route that minimizes the cost and the number of trees that have to be cut down for the side road. Draw a diagram of this route.  
       
     **Calculate the slope of AB using the coordinates of the campgrounds, A(2.0, 8.5) and B (10.0, 4.5)  
       
     Slope = Y2 – Y1**  
      **X2 – X1 = 4.5 – 8.5  
      10.0 – 2.0  
      = - 4.0  
      8.0  
      = - 0.5  
     Since the slope of AB = - 0.5, the slope of any line perpendicular to AB is  
     - 1 or 2.  
     -0.5**  
     **Now find equations for AB and RD b substituting the slope and the coordinates of a point into y = mx + b**

**For AB use A (2, 8.5) For RD, use R (6, 1.5)  
y = mx + b y = mx + b  
8.5 = -0.5 (2.0) + b 1.5 = 2 (6) + b  
8.5 = -1.0 + b 1.5 = 12 + b**

**9.5 = b - 10.5 = b  
  
An equation for AB is y = - 0.5x + 9.5 and an equation for RD is y = 2x – 10.5**

**Use substitution to find the point of intersection. Substitute x = 8 into the equation for either line to find the y coordinate.  
  
y = - 0.5x + 9.5 y = 2x – 10.5  
 = - 0.5 (8) + 9.5 = 2 (8) – 10.5  
 = - 4 + 9.5 = 16 – 10.5  
 = 5.5 = 5.5**  
 **POINT OF INTERSECTION FOR TWO ROADS IS (8.0, 5.5)**

* 1. Find the length of the side road, to the nearest tenth of a kilometer  
       
     **To calculate the length of a line segment RD, substitute the coordinates of its endpoints into the length formula.**

**RD = √ (X2 – X1)2 + (Y2 – Y1)2  
 = √ (8.0 – 6.0)2 + (5.5 – 1.5)2  
 = √ (2)2 + (4)2  
 = √ 4 + 16  
 = √ 20  
 = 4.5**

Example 2: Determine a Geometric Property Algebraically

The vertices of Triangle ABC are A (5, 5), B (-3, -1) and C (1, -3). Determine whether triangle ABC is a right triangle.

Draw a Diagram to help visualize the problem.

If triangle ABC is a right triangle, two of its sides are perpendicular to each other

Determine the slopes of the three sides of the triangle. Then, check is the product of any two of these slopes is -1

Or check the lengths of all three sides and check if the lengths satisfy the Pythagorean Theorem.

**Method 1: Use Slopes**

**Calculate the slope of each side of triangle ABC**

**SlopeAB = Y2 – Y1 SlopeBC = Y2 – Y1 SlopeAC = Y2 – Y1**

**X2 – X1 X2 – X1 X2 – X1  
 = -1 – 5 = -3 – (-1) = -3 – 5  
 -3 – 5 1 – (-3) 1 – 5  
  
 = - 6 = -3 + 1 = -8  
 - 8 1 + 3 - 4  
  
 = ¾ = - ½ = 2  
  
Since SlopeAC x SlopeBC = - 1, angle ACB is a right angle therefore Triangle ABC is a right triangle**

**Method 2 :Use the Pythagorean Theorem**